QCD FORTHE LHC

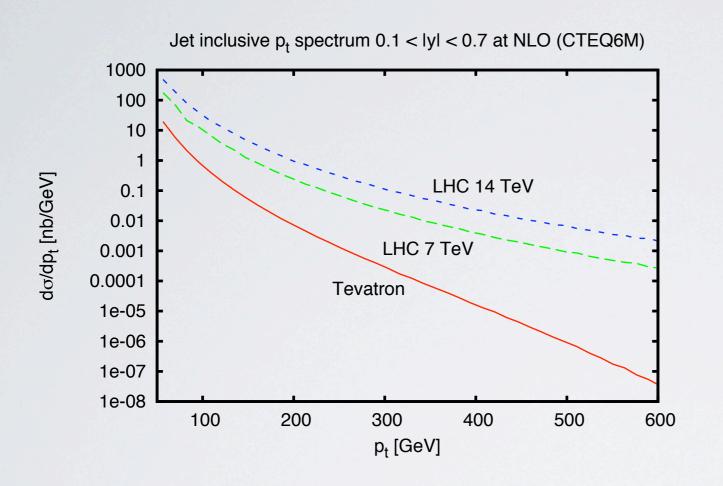
Babis Anastasiou ETH ZURICH

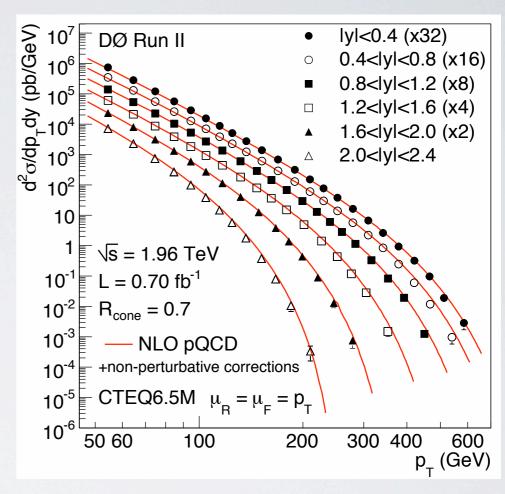
Brookhaven Forum 2010

OUTLINE

- The TEVATRON experience
- QCD for the sake of QCD
- QCD background
- QCD of new physics
- Theoretical breakthroughs and revelations

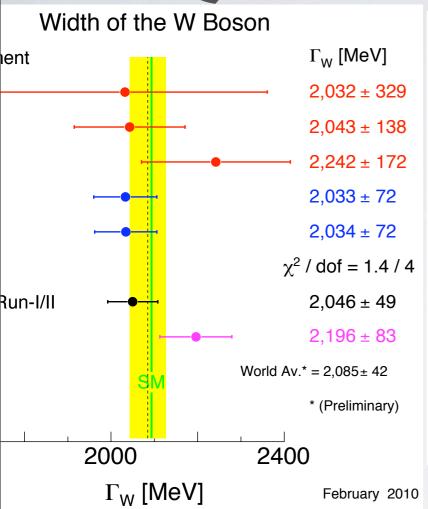
ENERGY WORLD RECORDS

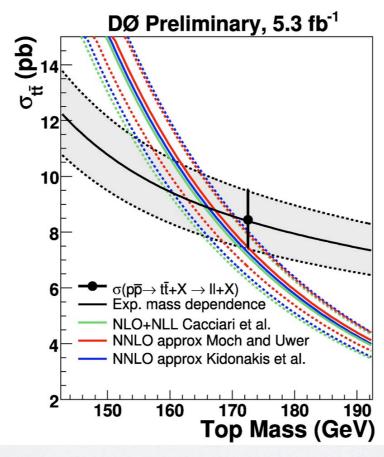


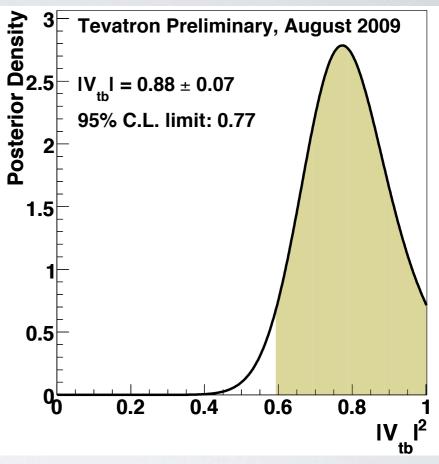


- Tevatron: plethora of data for QCD processes at very high energies.
- Detailed QCD analyses have been published.
- LHC: The next energy frontier, where a proof that QCD is a "domesticated" theory must be furnished.

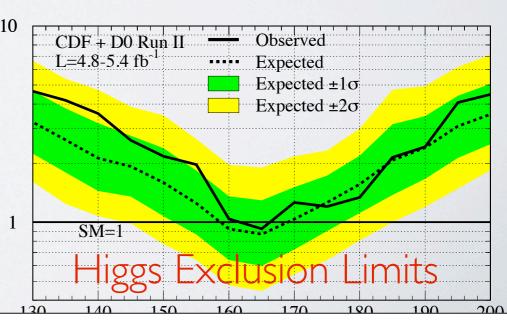
WHAT ARE THE FACES OF QCD AT THE TEVATRON?







- Precise QCD predictions are essential for almost every analysis.
- Progress in the understanding of high energy physics relies on QCD theory



QCD FOR THE LHC: "RETURN ON INVESTMENT"

- Precision determination of fundamental mass and coupling parameters and parton densities.
- Quantitative predictions for complicated backgrounds to the signatures of novel particles and interaction
- Efficient searches for new physics signals

- Reliable elimination of theoretical new physics models
- "Coronation" of the new physics paradigm after the Standard Model

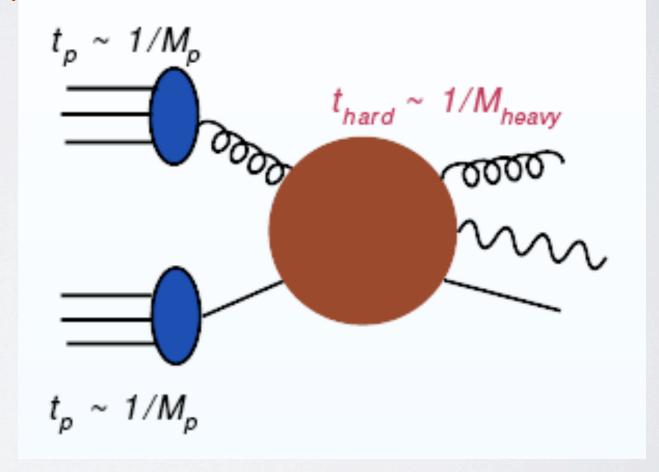
Understanding of the inner workings of gauge theories

 $ROI = \frac{Gain(Cost) - Cost}{Cost}$

FOUNDATIONS

QCD is a predictive theory

- Factorization
- Infrared Safety
- Perturbation theory
- (Global) experimental data

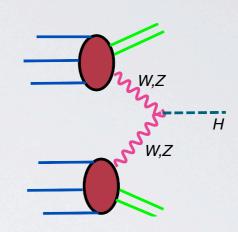


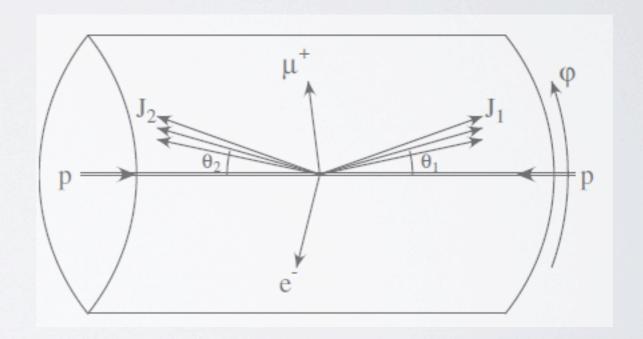
but not a solved theory!

EXAMPLE: WEAK BOSON FUSION

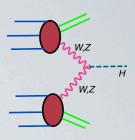
- Characteristic topology with a pair of two forward jets.
- Little color exchange in the t-channel, little amount of radiation from perturbative QCD in central detector regions

(Rainwater, Zeppenfeld; Rainwater, Zeppenfeld, Hagiwara; Plehn, Rainwater, Zeppenfeld; ...)

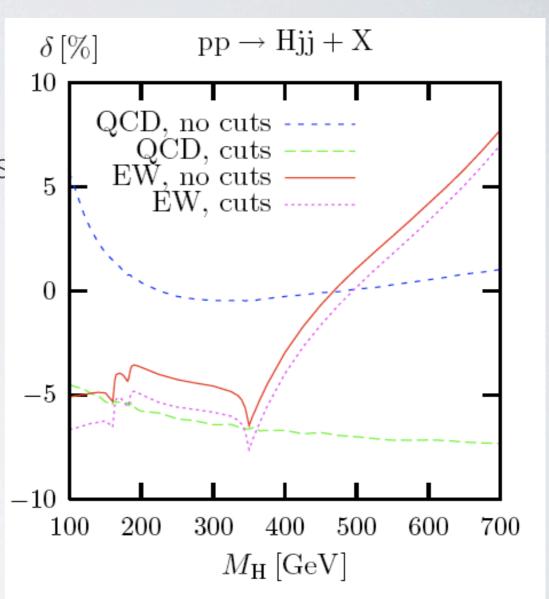




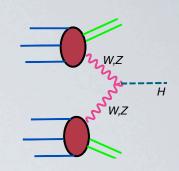
PERTURBATIVE EFFECTS



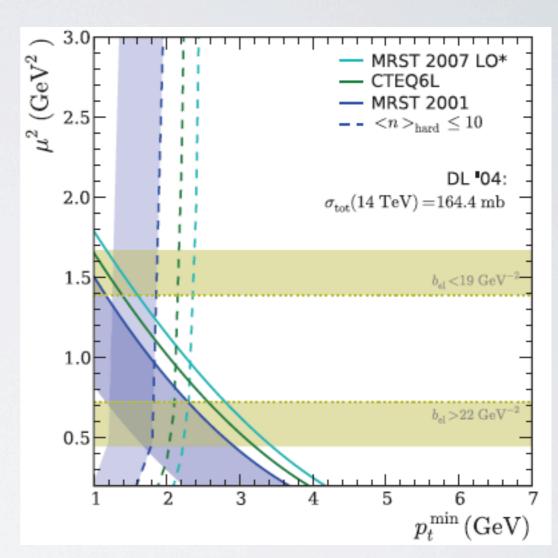
- NLO QCD (Han, Valencia, Willenbrock;
 Figy, Oleari, Zeppenfeld; Berger, Campbell)
- NLO QCD and electro-weak corrections (Ciccolini, Denner, Dittmaier)
- Signal-Background interference (Andersen, Binoth, Heinrich, Smillie)
- Gluon induced weak boson fusion (Harlander, Vollinga, Weber)
- Total cross-section in NNLO QCD and 2% estimated
 precision (Bolzoni, Maltoni, Moch, Zaro)



NON PERTURBATIVE QCD EFFECTS



- no central jets with Pt > 20
 GeV, sensitive to the underlying event
- we shall need to revisit
 underlying-event models at the
 LHC (Baehr, Butterworth, Seymour;
 Baehr, Gieseke, Seymour;
 Dasgupta, Magnea, Salam;...)
- also, revisit jet-veto analysis after first (7 TeV) and second (14 TeV) LHC data.

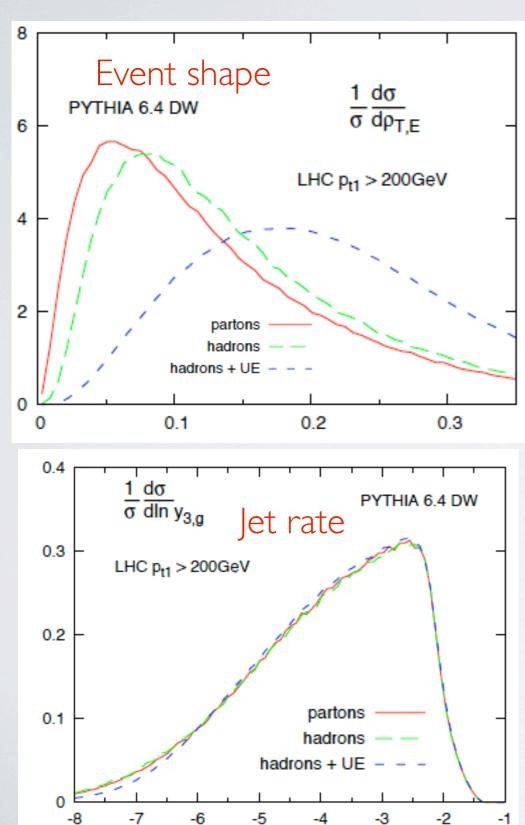


Baehr, Butterworth, Seymour:
restrictions from LHC
total cross-section
on a eikonal model for the UE

ANALYZINGTHE MAKE OUT OF JETS

- · Jets are rich in their topology.
- Contain information on their origin (QCD low or high-pt splittings, decays of colorful or colorless heavy particles, etc)
- Jet definitions and observables can be a powerful tool for LHC studies
- · Event shapes probe the anatomy of QCD radiation.

EVENT SHAPES AT HADRON COLLIDERS AND NON-PERTURBATIVE EFFECTS



Banfi, Salam, Zanderighi

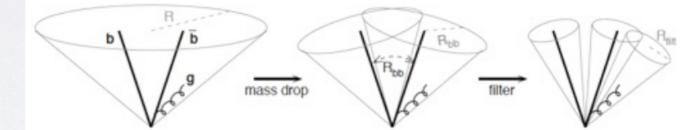
- Jet resolution and event shape variables have different sensitivity to hadronization and underlying event
- Can be used to tune parton-shower Monte-Carlo's at the LHC.

JET SUBSTRUCTURE

Butterworth, Davison, Rubin, Salam

 Check for events where the higgs and the vector boson are back-to-back

cluster into fat jets. analyze their make up



₩ qq

→V+jets

V+Higgs

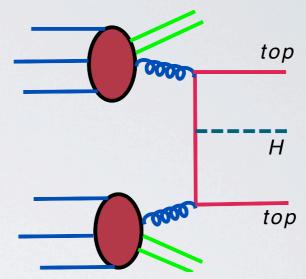
Mass (GeV)

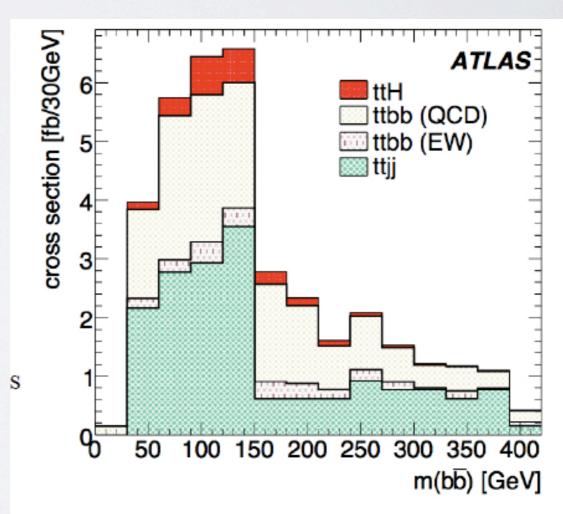
- two b-tagged smaller
 size jets with roughly same mass?
- filter underlying event with a smaller jet-size parameter (R)

Jet definition	$\sigma_S/{ m fb}$	$\sigma_B/{ m fb}$	$S/\sqrt{B \cdot \mathrm{fb}}$
C/A, $R = 1.2$, $MD-F$	0.57	0.51	0.80
$K_{\perp}, R = 1.0, y_{cut}$	0.19	0.74	0.22
SISCone, $R = 0.8$	0.49	1.33	0.42

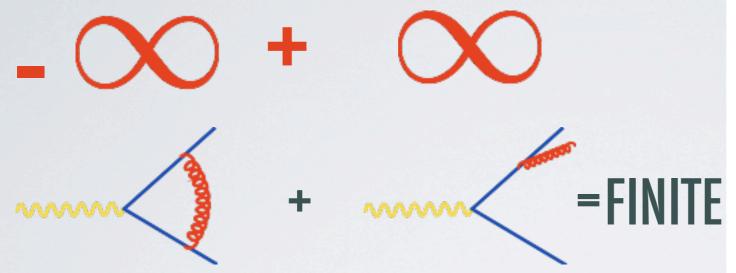
ASSOCIATED WITH TOP

- Direct access to the top yukawa coupling
- Large backgrounds, difficult combinatorics (six jets)
- dropped out from the list of discovery channels
- can revive it with "jet tomography"

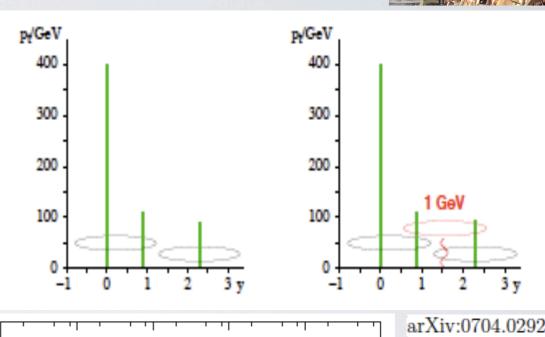


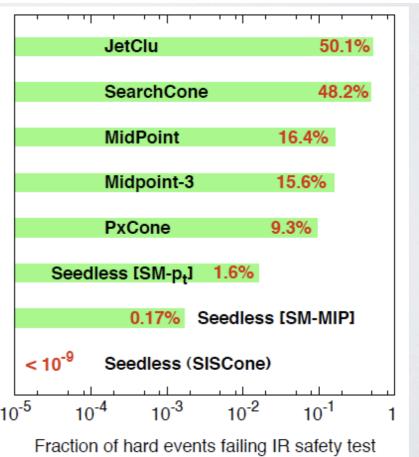


JETS AND INFRARED SAFETY



- Soft or Collinear parton emission must not alter the number of jets in an event.
- Many jet measurements are not directly comparable to perturbative calculations (e.g. W+3 jets with JETCLU @ NLO)
 - infrared safe algorithms

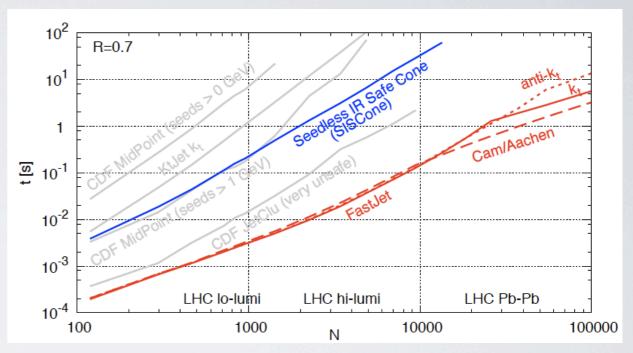


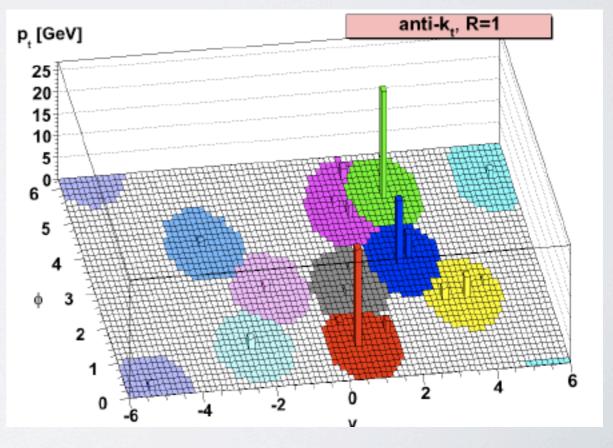


FAST AND SAFE JET FINDING

Cacciari, Salam, Soyez (2007-2009)

- Fast implementation of recombination algorithms
- New infrared safe cone algorithm (SISCone)
- Better understanding of jet areas
- anti-Kt: recombination algorithm with "perfect cones"

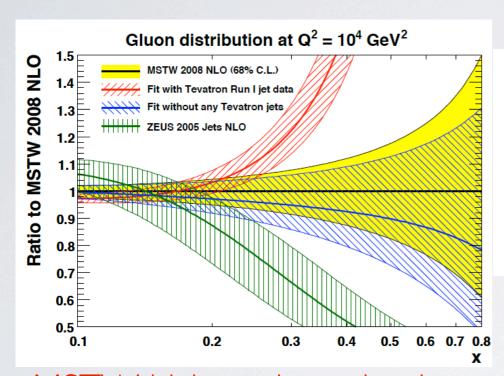




PARTON DENSITIES

- Several efforts for a precise determination of parton densities:
 CTEQ: Pumplin, Huston, Lai, Nadolsky, Tung, Yuan(NLO, global fit);
 MSTW: Martin, Stirtling, Thorne, Watt (NNLO, global fit);
 JR: Jimenez-Delgado, Reya (NNLO, DIS fit);
 ABKM: Alehkin, Bluemlein, Klein, Moch (NNLO, DIS and Drell-Yan fit);
 HERA colloborations (NNLO, DIS fit)
- Input for precise hadron collider phenomenology.
- New ideas on pdf extraction, using Artificial Neural Network methods (Ball, Del Debbio, Forte, Guffanti, Latorre, Piccione, Rojo, Ubialli)
- Improvements on theoretical treatment, better error estimation, but also important changes from older sets

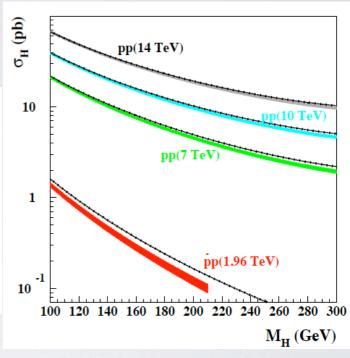
IMPORTANT PDF DIFFERENCES



MSTW high-x gluon density, impact of Tevatron jet measurements

\sqrt{s} (TeV)	ABKM	MSTW2008
1.96 (<i>p̄p</i>)	6.91 ± 0.17	7.04
7 (<i>pp</i>)	131.3 ± 7.5	160.5
10 (pp)	343 ± 15	403
14 (<i>pp</i>)	780 ± 28	887

MSTW vs ABKM for top pair cross-section



MSTW vs ABKM for Higgs cross-section

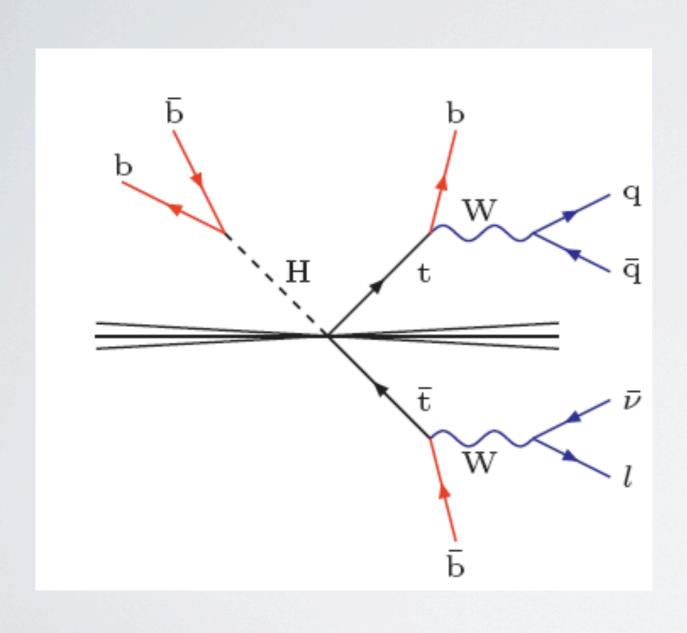
MRST 2001	MRST 2004	MRST 2006	MSTW 2008
0.3833	0.3988	0.3943	0.3444

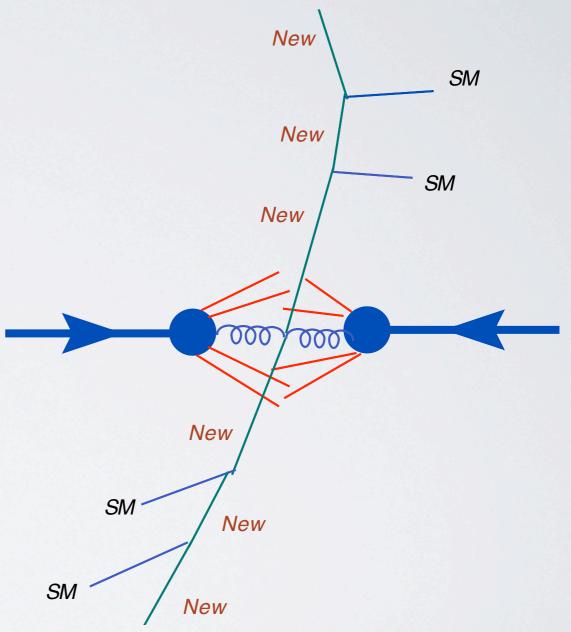
LHC data and QCD theory will be very useful to constrain pdfs

A difficult case: high-x gluon densities

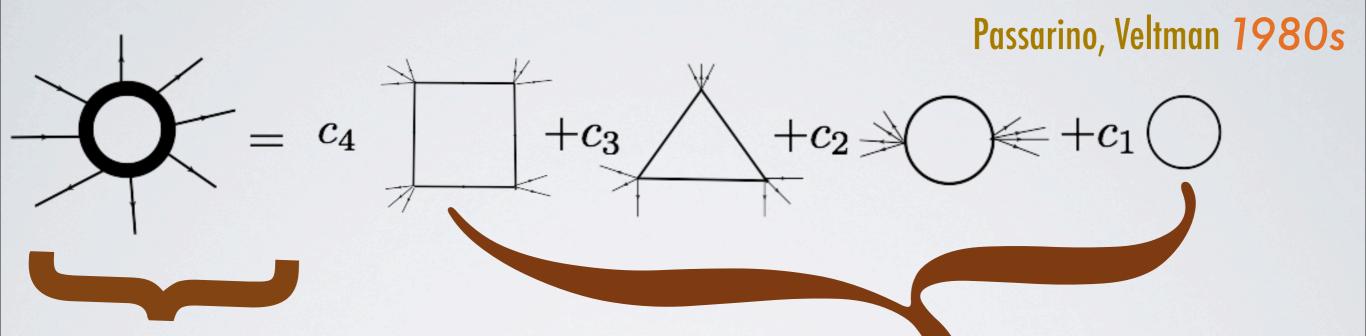
Higgs cross-section at the TEVATRON MSTW vs MRST

NEW PHYSICS APPETITE FOR COMPLICATED QCD SIMULATIONS





MASTER INTEGRALS



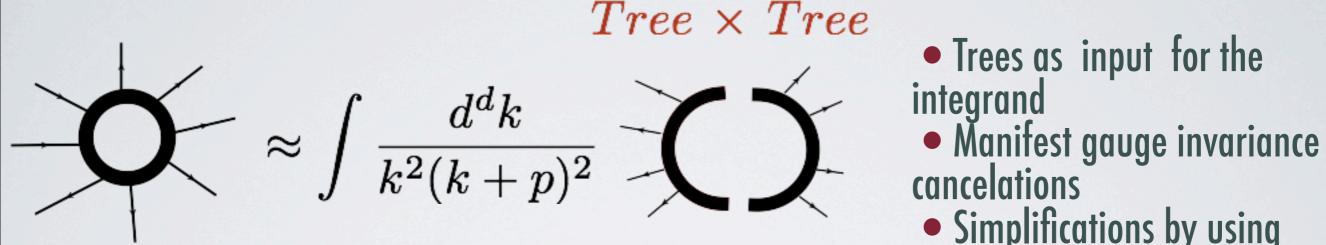
One-loop amplitude in Gauge theory

Integrals in scalar field theory

Known method(s) to compute a,b,c,d coefficients had a (# Legs)! computational cost

UNITARITY: A VISIONARY IDEA

Bern, Dixon, Dunbar, Kosower 1990s



Trees as input for the

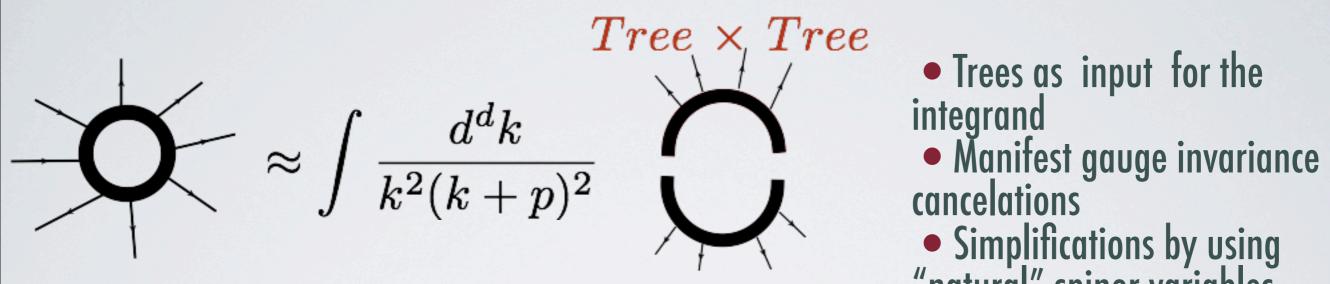
Simplifications by using "natural" spinor variables

Mismatch between Trees in four dimensions and loop integration in D-dimensions
 Introduction of four dimensional helicity regularization scheme
 Clever theory input (collinear factorization) to recover the full one-loop amplitude

Trees were an essential ingredient. No explicit connection of master integral coefficients to tree amplitudes.

UNITARITY: A VISIONARY IDEA

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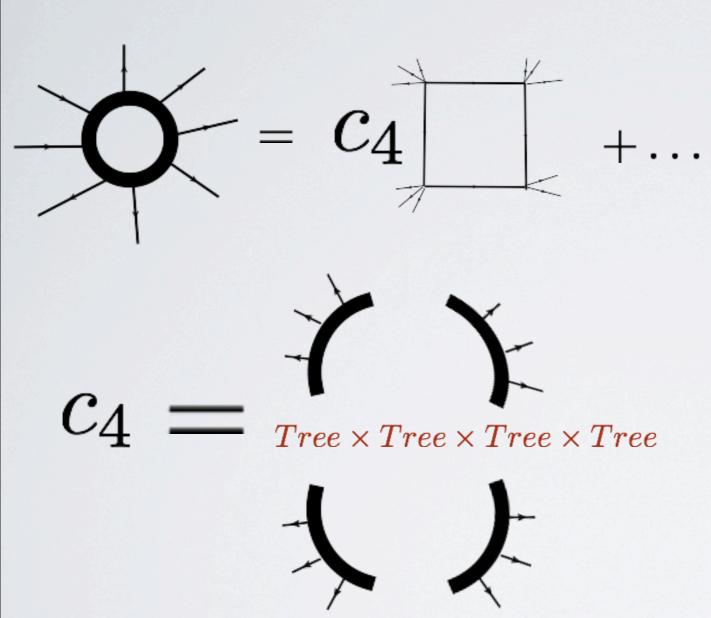
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COEFFICIENT OF BOX MASTER

Britto, Cachazo, Feng 2004



- Simple product of four tree amplitudes
- Evaluated at complex momenta
- corresponding to loop momentum values where all propagators of the box master integral are ON-SHELL

Ossola, Papadopoulos, Pittau 2006

(building on del Aguila, Pittau, 2004)

(building on del Aguila, Pittau, 2004)
$$=\int \frac{d^dk}{(2\pi)^d} \left[c_4 f_4(\vec{k}) + c_3 f_3(\vec{k}) + c_2 f_2(\vec{k}) + c_1 f_1(\vec{k}) \right]$$

$$+\tilde{c}_4 \, \tilde{f}_4(\vec{k}) + \tilde{c}_3 \, \tilde{f}_3(\vec{k}) + \tilde{c}_2 \, \tilde{f}_2(\vec{k}) + \tilde{c}_1 \, \tilde{f}_1(\vec{k})$$

Ossola, Papadopoulos, Pittau 2006

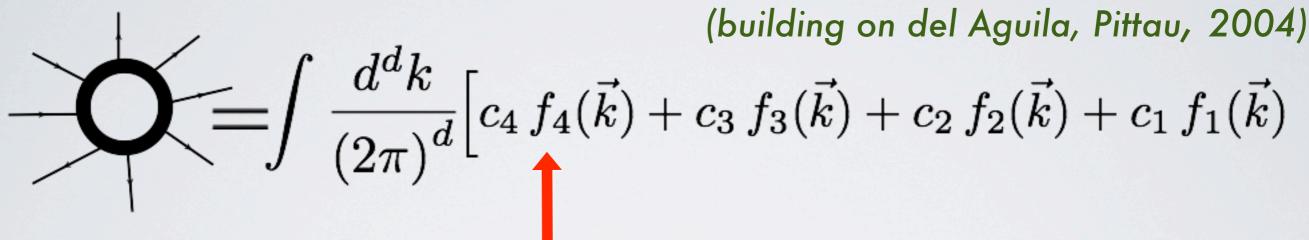
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$$= \int \frac{d^dk}{(2\pi)^d} \left[c_4 f_4(\vec{k}) + c_3 f_3(\vec{k}) + c_2 f_2(\vec{k}) + c_1 f_1(\vec{k}) \right]$$

$$+ \tilde{c}_4 \tilde{f}_4(\vec{k}) + \tilde{c}_3 \tilde{f}_3(\vec{k}) + \tilde{c}_2 \tilde{f}_2(\vec{k}) + \tilde{c}_1 \tilde{f}_1(\vec{k}) \right]$$
After Integration:

After Integration:

Ossola, Papadopoulos, Pittau 2006

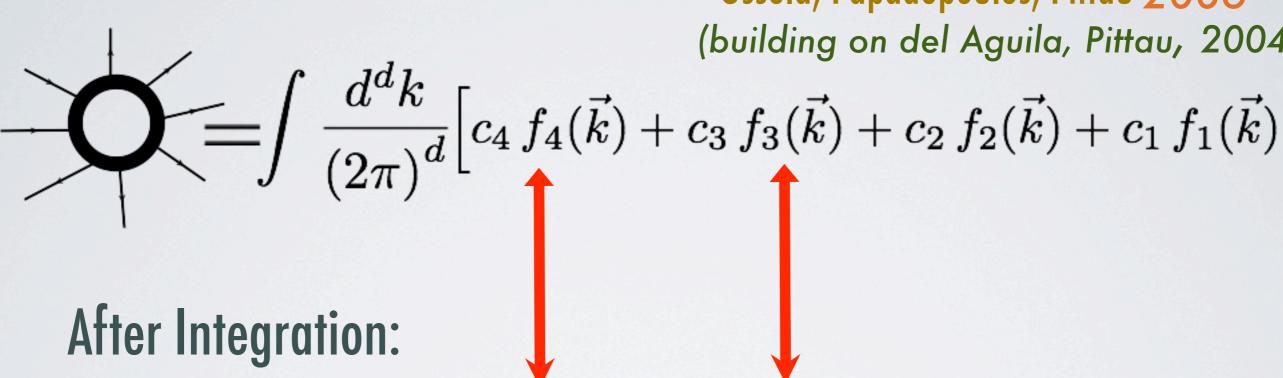


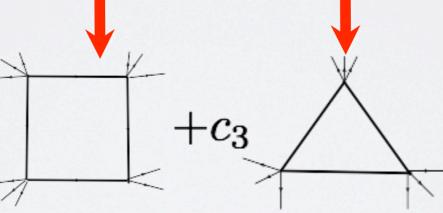
After Integration:

$$= c_4$$

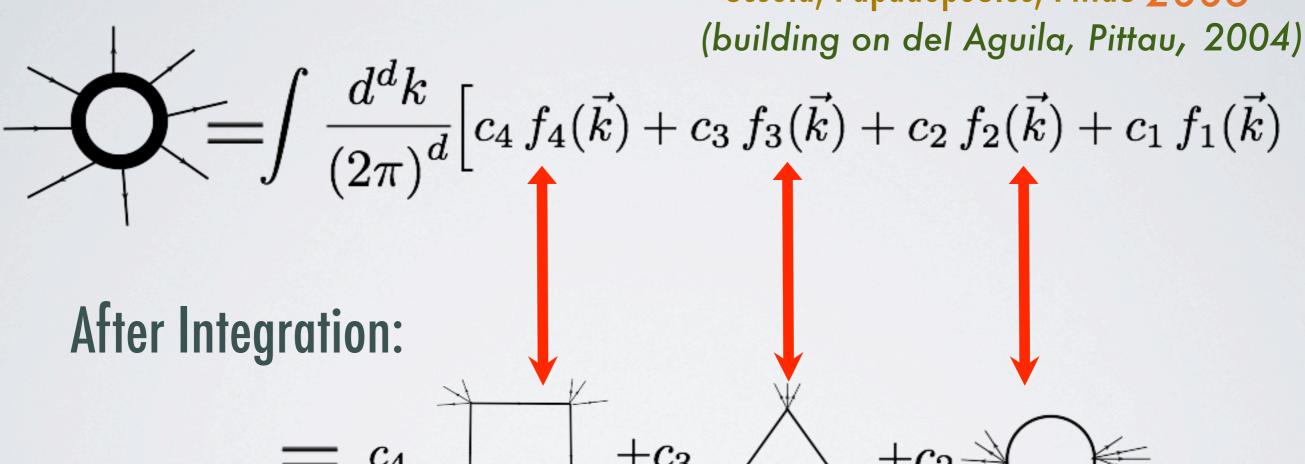
Ossola, Papadopoulos, Pittau 2006

(building on del Aguila, Pittau, 2004)

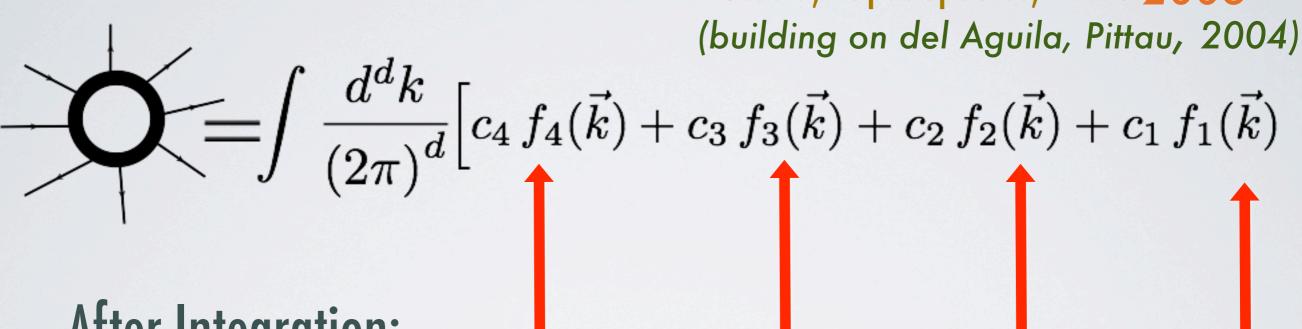




Ossola, Papadopoulos, Pittau 2006



Ossola, Papadopoulos, Pittau 2006



After Integration:

$$= c_4 + c_3 + c_2 + c_1$$

Ossola, Papadopoulos, Pittau 2006

$$= \int \frac{d^dk}{(2\pi)^d} \left[c_4 f_4(\vec{k}) + c_3 f_3(\vec{k}) + c_2 f_2(\vec{k}) + c_1 f_1(\vec{k}) + \tilde{c}_4 \tilde{f}_4(\vec{k}) + \tilde{c}_3 \tilde{f}_3(\vec{k}) + \tilde{c}_2 \tilde{f}_2(\vec{k}) + \tilde{c}_1 \tilde{f}_1(\vec{k}) \right]$$

Ossola, Papadopoulos, Pittau 2006

$$= \int \frac{d^dk}{(2\pi)^d} \left[c_4 f_4(\vec{k}) + c_3 f_3(\vec{k}) + c_2 f_2(\vec{k}) + c_1 f_1(\vec{k}) + \tilde{c}_4 \tilde{f}_4(\vec{k}) + \tilde{c}_3 \tilde{f}_3(\vec{k}) + \tilde{c}_2 \tilde{f}_2(\vec{k}) + \tilde{c}_1 \tilde{f}_1(\vec{k}) \right]$$

 $\tilde{f}_i(\vec{k}), f_i(\vec{k})$: Known rational functions of the loop momentum

Ossola, Papadopoulos, Pittau 2006

$$= \int \frac{d^dk}{(2\pi)^d} \left[c_4 f_4(\vec{k}) + c_3 f_3(\vec{k}) + c_2 f_2(\vec{k}) + c_1 f_1(\vec{k}) + \tilde{c}_4 \tilde{f}_4(\vec{k}) + \tilde{c}_3 \tilde{f}_3(\vec{k}) + \tilde{c}_2 \tilde{f}_2(\vec{k}) + \tilde{c}_1 \tilde{f}_1(\vec{k}) \right]$$

 $\tilde{f}_i(\vec{k}), f_i(\vec{k})$: Known rational functions of the loop momentum

 \tilde{c}_i, c_i : coefficients can be determined algebraically computing the integrand at a sufficient number of values for \vec{k}

Ossola, Papadopoulos, Pittau 2006

$$\int \frac{d^d k}{(2\pi)^d} \left[c_4 f_4(\vec{k}) + c_3 f_3(\vec{k}) + c_2 f_2(\vec{k}) + c_1 f_1(\vec{k}) + \tilde{c}_4 \tilde{f}_4(\vec{k}) + \tilde{c}_3 \tilde{f}_3(\vec{k}) + \tilde{c}_2 \tilde{f}_2(\vec{k}) + \tilde{c}_1 \tilde{f}_1(\vec{k}) \right]$$

Ossola, Papadopoulos, Pittau 2006

$$\int \frac{d^dk}{(2\pi)^d} \left[c_4 f_4(\vec{k}) + c_3 f_3(\vec{k}) + c_2 f_2(\vec{k}) + c_1 f_1(\vec{k}) + \tilde{c}_4 \tilde{f}_4(\vec{k}) + \tilde{c}_3 \tilde{f}_3(\vec{k}) + \tilde{c}_2 \tilde{f}_2(\vec{k}) + \tilde{c}_1 \tilde{f}_1(\vec{k}) \right] = \int \frac{d^dk}{(2\pi)^d}$$

Integrand is "easy", essentially a tree amplitude

Ossola, Papadopoulos, Pittau 2006

$$\int \frac{d^dk}{(2\pi)^d} \left[c_4 f_4(\vec{k}) + c_3 f_3(\vec{k}) + c_2 f_2(\vec{k}) + c_1 f_1(\vec{k}) + \tilde{c}_4 \tilde{f}_4(\vec{k}) + \tilde{c}_3 \tilde{f}_3(\vec{k}) + \tilde{c}_2 \tilde{f}_2(\vec{k}) + \tilde{c}_1 \tilde{f}_1(\vec{k}) \right] = \int \frac{d^dk}{(2\pi)^d}$$

Integrand is "easy", essentially a tree amplitude

Evaluate integrand at loop momenta values such as loop particles are set ON SHELL

Ossola, Papadopoulos, Pittau 2006

$$\int \frac{d^dk}{(2\pi)^d} \left[c_4 f_4(\vec{k}) + c_3 f_3(\vec{k}) + c_2 f_2(\vec{k}) + c_1 f_1(\vec{k}) + \tilde{c}_4 \tilde{f}_4(\vec{k}) + \tilde{c}_3 \tilde{f}_3(\vec{k}) + \tilde{c}_2 \tilde{f}_2(\vec{k}) + \tilde{c}_1 \tilde{f}_1(\vec{k}) \right] = \int \frac{d^dk}{(2\pi)^d}$$

Integrand is "easy", essentially a tree amplitude

Evaluate integrand at loop momenta values such as loop particles are set ON SHELL

ON-SHELL: determines coefficients successively

COEFFICIENTS AS TREE PRODUCTS

Ellis, Giele, Kunszt 2007

- ON-SHELL loop propagators = Product of tree amplitudes
- Evaluation of trees with powerful recursive methods

e.g. Berends-Giele, Britto-Cachazo-Feng-Witten, etc

CONFLICT OF DIMENSIONS

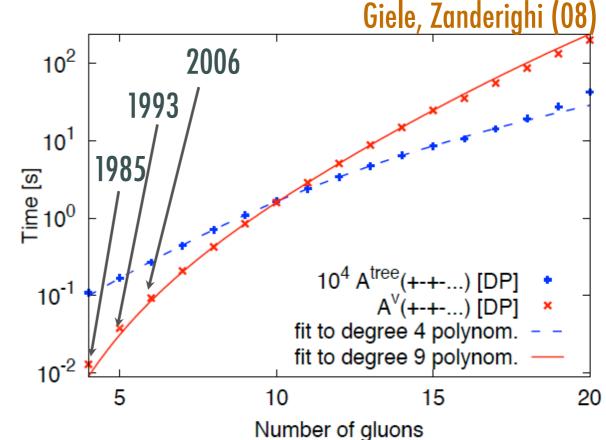
Loop Integrations in D dimensions, Tree amplitudes in four dimensions. Mismatch, i.e. missing terms from amplitude evaluation. Requires a second calculation.

- Specialized tree-like recursions in D=4 for the missing terms
 Berger, Bern, Dixon, Forde, Kosower 2006
- Elegant/general solution: Amplitude in a general dimension from results in D=5 and D=6. Ellis, Giele, Kunszt, Melnikov 2008
- Specialized Feynman rules for missing terms:
 Draggiotis, Garzelli, Papadopoulos, Pittau 2009

BREATHTAKING DEVELOPMENTS

One-loop amplitudes with 22 gluons Giele, Zanderighi (08); Lazopoulos (08); Giele, Winter (09)

numerical evaluation of all 2 to 4 amplitudes in the Les-Houches 2007 wish-list $a\bar{a} \rightarrow t\bar{t}h\bar{h} h\bar{h}h$



Houches 2007 van Hameren, Papadopoulos, Pittau (09) $qar{q}, gg \to tar{t}bar{b}, bar{b}bar{b}, W^+W^-bar{b}, tar{t}gg$ $qar{q}' \to Wggg, Zggg$

NLO CALCULATIONS @ LHC

- What can we hope for?
- We cannot do better than tree calculations..., i.e. processes with 7 or 8 particles in the final state.
- All 2 to 4 processes with both Feynman diagrammatic and unitarity methods
- 2 to 5 and perhaps 2 to 6 processes with unitarity methods

(2 to 4) HADRON COLLIDER PROCESSES

$$pp \to t\bar{t}b\bar{b}$$

Bredenstein, Denner, Dittmaier, Pozzorini Bevilacqua, Czakon, Papadopoulos, Worek

$$pp \to t\bar{t}jj$$

Bevilacqua, Czakon, Papadopoulos, Worek

$$pp \to W^{\pm} + 3jets$$

Berger, Bern, Dixon, Febres Cordero, Forde, Gleisberg, Ita, Kosower, Maitre Ellis, Kunszt, Melnikov, Zanderighi

$$pp \rightarrow Z + 3jets$$

Berger, Bern, Dixon, Febres Cordero, Forde, Gleisberg, Ita, Kosower, Maitre

$$pp \rightarrow W + 4jets$$
 (first results)

Berger, Bern, Dixon, Febres Cordero, Forde, Gleisberg, Ita, Kosower, Maitre

LESSONS FROM MULTILEG NLO CALCULATIONS

- Guessing higher order corrections for multi-particle background processes without explicit calculations is hopeless
- There exists no unique "K-factor"
 "across the full phase-space for
 processes with such
 complicated dynamics
- NLO computations can be used to optimize LO Monte-Carlo's

SUSY BACKGROUND

 $pp \to W(\to \tau \nu) + 3jets$

ATLAS CUTS: $\sigma_{NLO} \simeq 200\% \sigma_{LO}$

CMS CUTS: $\sigma_{NLO} \simeq 110\% \sigma_{LO}$

Melnikov, Zanderighi

e.g. local scale for alphas in each branching

THE NNLO FRONT

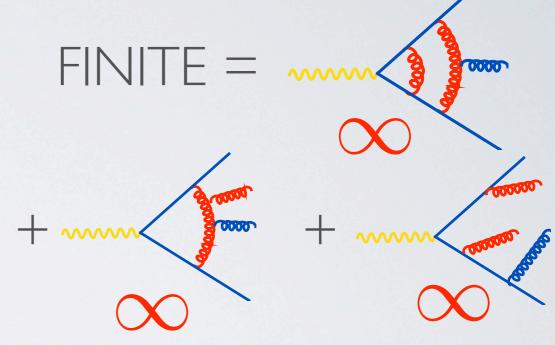
- Precision of measurements at collider experiments is often excellent
- Perturbation theory is often slow at work, first correction after the leading order too large and too uncertain.
- All "2 to 1" and "2 to 2" hadron collider processes must be computed at NNLO.
- LEP, HERA, TEVATRON, LHC data = NNLO phenomenology

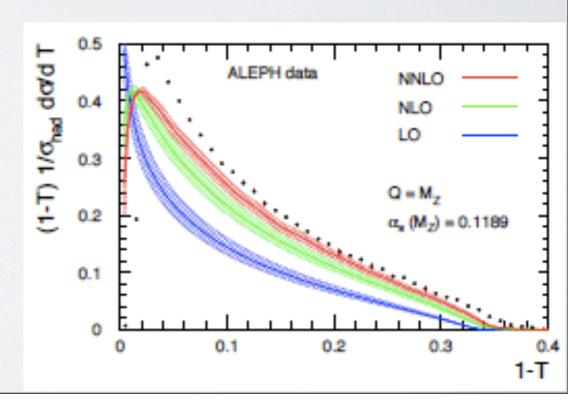
THREE-JET EVENTS FROM LEP

- LEP Legacy: Excellent measurements of three jet crosssections and jet event shapes at various energies.
- Precise extraction of the strong coupling constant; largest error from theoretical prediction of the cross-section.
- •NNLO corrections to $e^+e^- \to 3jets$ was the holy grail of the QCD community for more than a decade.

CANCELATION OF SINGULARITIES

- Two-loop amplitude computed already in 2001 by Garland, Gehrmann, Glover, Koukoutsakis, Remiddi
- A universal method for the cancelation of matrix element singularities through NNLO for lepton collider processes by Gehrmann-de Ridder, Gehrmann, Glover, Heinrich (2007)
- Revision by Weinzierl (2008).





Os FROM JET EVENT SHAPES

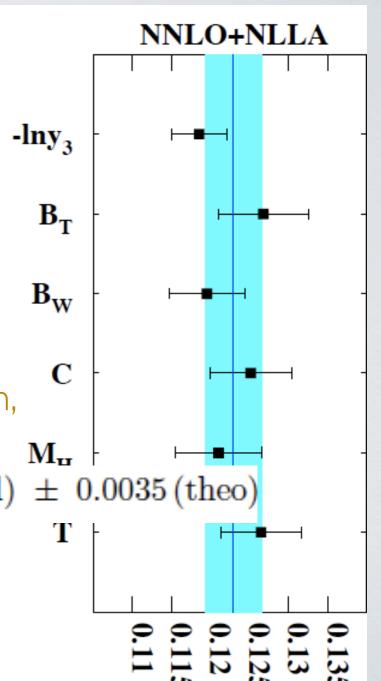
 A synthesis of fixed order QCD, Electroweak corrections, resummation, and hadronization effects describe excellently three jet events at LEP.

 State of the art extraction of alphas with the NNLO result + NLL resummation Dissertori, Gehrmann-de Ridder, Gehrmann, Glover, Heinrich, Luisoni, Stenzel

 $\alpha_s(M_{\rm Z}) = 0.1224 \pm 0.0009 \,({\rm stat}) \pm 0.0009 \,({\rm exp}) \pm 0.0012 \,({\rm had})$

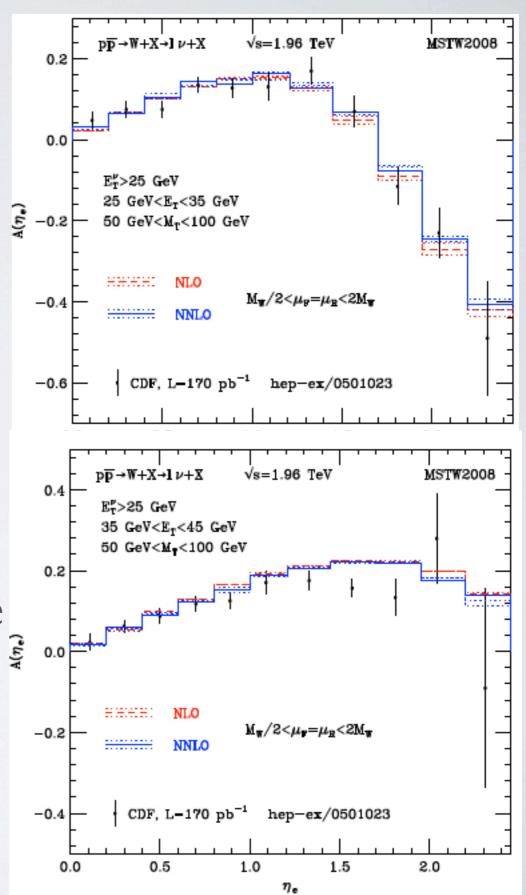
• also from NNLO+"SCET resummation" of the thrust distribution (Becher, Schwarz).

arXiv:0906.3436



DRELL-YANTHEORY

- NNLO total cross-section Hamberg, van Neerven (1990); Harlander, Kilgore (2002)
- NNLO rapidity distribution CA,Dixon,Menikov,Petriello (2004)
- Fully differential NNLO
 Melnikov, Petriello (2006);
 Catani, Cieri, Ferrera, Grazzini (2009)
- Recent application, lepton charge asymmetry
 Catani, Ferrera, Grazzini (2010)

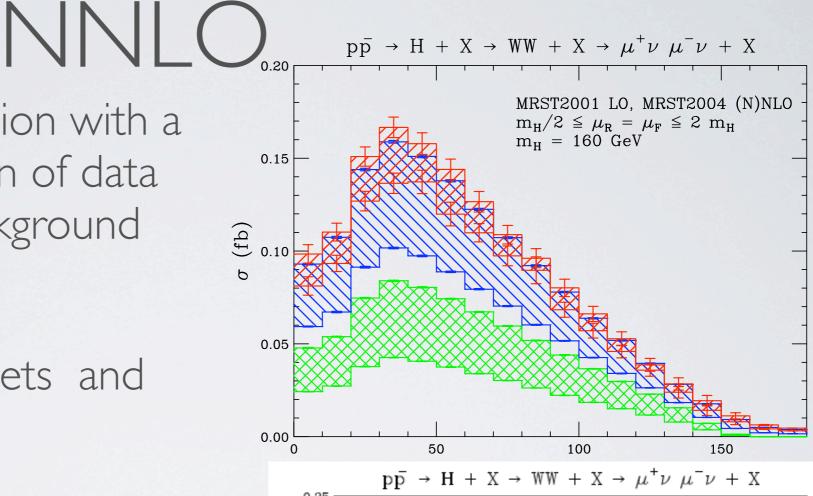


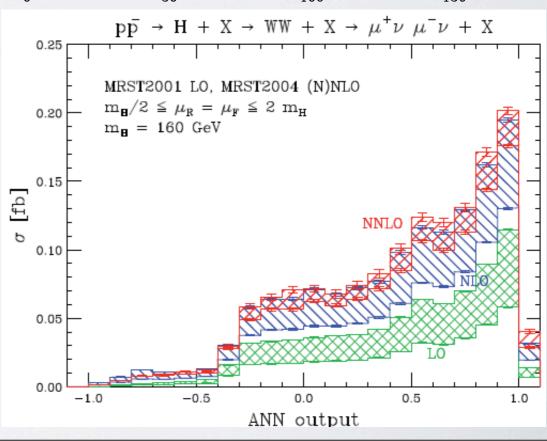
HIGGS PRODUCTION AT

 TEVATRON exclusion with a detailed comparison of data with signal and background distributions

 important cuts on jets and lepton isolation

- Fully Exclusive Higgs Production (CA,Melnikov,Petriello; CA, Dissertori, Stoeckli)
- HNNLO method (Catani, Grazzini;
 Grazzini)

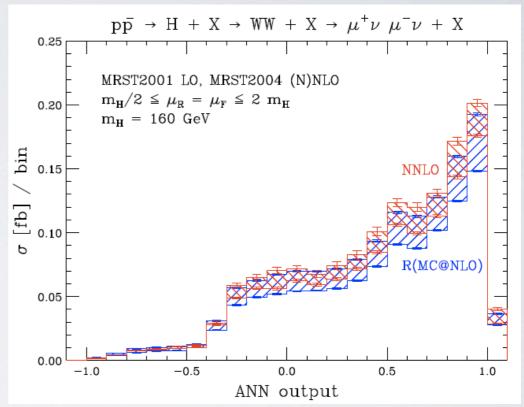




GENERATORS DIFFER

- PYTHIA has a smaller jet-veto and isolation acceptance than HERWIG and MC@NLO
- HERWIG and MC@NLO closer to NNLO

VALIDATION is indispensable!



(CA, Dissertori, Grazzini, Stoeckli, Webber)

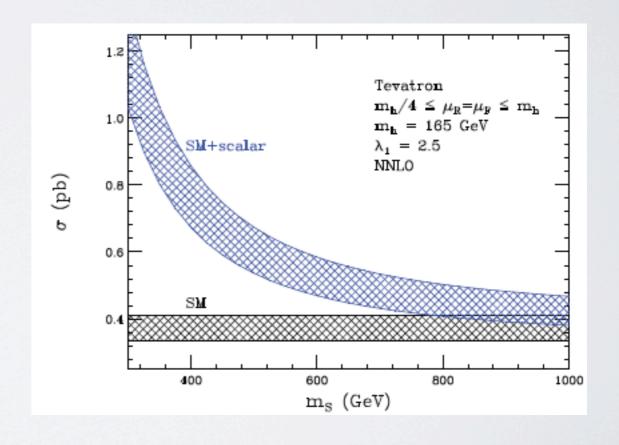
$\sigma_{ m acc}/\sigma_{ m incl}$	Trigger	+ Jet-Veto	+ Isolation	All Cuts
NNLO $(\mu = m_{\rm H}/2)$	44.7%	39.4% (88.1%)	36.8% (93.4%)	27.8% (75.5%)
NNLO ($\mu = 2 m_{\rm H}$)	44.9%	41.8% (93.1%)	40.7% (97.4%)	31.0% (76.2%)
MC@NLO ($\mu = m_{\rm H}/2$)	44.4%	38.1% (85.8%)	35.3% (92.5%)	26.5% (75.2%)
MC@NLO ($\mu = 2 m_{\rm H}$)	44.8%	38.8% (86.7%)	35.9% (92.5%)	27.0% (75.2%)
HERWIG	46.7%	40.8% (87.4%)	37.8% (92.7%)	28.6% (75.7%)
PYTHIA	46.6%	37.9% (81.3%)	32.2% (85.0%)	24.4% (75.8%)

BEYOND THE STANDARD GLUON FUSION

- Can we derive a mass exclusion limit for a BSM scalar Higgs boson from an experimental analysis based on SM theoretical predictions?
- very often yes, if QCD corrections and shapes of signal discriminants are model independent.
- CAN WE USE EXPERIMENTAL LIMITS OR A DISCOVERY AS PRECISION TESTS?
- Until recently no complete NNLO calculation for any extension of the SM (not even a fourth quark generation)

BSM HIGGS PRODUCTION AT NNLO

- Additional heavy quark families (CA, Boughezal, Furlan)
- Colour octet scalars (Boughezal, Petriello)



FUTURE NNLO PHENOMENOLOGY

- We need to develop methods that can be used for 2 to 2 scattering processes.
- Top-pair production, Di-boson production, and other routine processes will be simulated with high precision
- A big theoretical challenge, which requires additional efforts

RESUMMATION

- Progress in matching parton-showers and NLO calculations (MC@NLO:Webber,Frixione; White,Frixione,Laenen,Maltoni POWEHEG:Frixione,Nason,Oleari; Aliole,Nason,Oleari,Re;)
- Resummation in SCET
 thrust in ee, inclusive photons: Becher, Schwarz
 Drell-Yan and Higgs: Idilbi, Xi, Yuan, Ahrens, Becher, Neubert
 top-pair NLO+NNLL: Ahrens, Ferroglia, Neubert, Pecjak, Yang
 also Czakon, Mitov, Sterman

ITERATIVE PERTURBATION SERIES

- The perturbation series of gauge theories displays cross-order iterations.
- These are needed to cancel infrared and UV divergences, filtering the superposition principle from ultra short and very large distance effects.
- They are exploited to formulate parton shower algorithms, and resumming large logarithms.
- But, the remainder seems very different at each order in perturbation theory!

AN UNEXPECTED ITERATION IN N=4 SUPER YANG-MILLS THEORY

$$\mathcal{M}_{4}^{(2)}(\epsilon) = \frac{1}{2} \left(\mathcal{M}_{4}^{(1)}(\epsilon) \right)^{2} + f^{(2)}(\epsilon) \mathcal{M}_{4}^{(1)}(2\epsilon) + C^{(2)} + \mathcal{O}(\epsilon)$$

CA, Bern, Dixon, Kosower

$$\mathcal{M}_{n}^{(2)}(\epsilon) = \frac{1}{2} \left(\mathcal{M}_{n}^{(1)}(\epsilon) \right)^{2} + f^{(2)}(\epsilon) \mathcal{M}_{n}^{(1)}(2\epsilon) + C^{(2)} + \mathcal{O}(\epsilon)$$

$$\mathcal{M}_{4}^{(3)}(\epsilon) = -\frac{1}{3} \left(\mathcal{M}_{4}^{(1)}(\epsilon) \right)^{3} + \mathcal{M}_{4}^{(2)}(\epsilon) \mathcal{M}_{4}^{(1)}(\epsilon) + f^{(1)}(\epsilon) \mathcal{M}_{4}^{(1)}(3\epsilon) + C^{(3)} + \mathcal{O}(\epsilon)$$

Bern, Dixon, Smirnov

Can be computed in the strong limit with AdS/CFT Alday, Maldacena

$$\mathcal{M}_n = \exp \left[\sum_{l=1}^{\infty} a^l f^{(l)}(\epsilon) \mathcal{M}_n^{(1)}(l\epsilon) + C^{(l)} + \mathcal{O}(\epsilon) \right]$$

$$\ln(1+\sum_{l=1}^{\infty}a^{l}\mathcal{M}_{n}^{(l)})=\ln(1+\sum_{l=1}^{\infty}a^{l}W_{n}^{(l)})+\mathcal{O}(\epsilon)$$

<Wilson Loop> = Amplitude Sokachev,Korchemsky

Can compute two-loop amplitudes with arbitrary number of CA,Brandhuber,Heslop,Khoze,Spence,Travaglini legs, using the Wilson-loop duality

One-loop amplitudes from trees... and masters!!!



Trees in Gauge theory



Loop Master Integrals in scalar field theory

OUTLOOK

- Our abilities in simulating precisely collider processes have grown tremendously.
- New computational methods at NLO are extremely powerful. A classic work which will be part of future field theory books.
- Ready to take on the big challenge of finding new physics convincingly in hadron collider data.